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Tessellations of hyperbolic 3-space and Bloch groups

July 5, 2019, 14:30-15:30

Let k be an imaginary quadratic number field with ring of integers R. We discuss how an ideal tessellation of hyperbolic 3-space on which $\mathrm{GL}_2(R)$ acts gives rise to an explicit element b of infinite order in the second Bloch group for k, and hence to an element c in $\mathrm{K}_3(k)$ modulo torsion, which is cyclic of infinite order. The regulator of c equals $-24\zeta_k'(-1)$, and the Lichtenbaum conjecture for k at -1 implies that a generator of $\mathrm{K}_3(k)$ modulo torsion can be obtained by dividing c by twice the order of $\mathrm{K}_2(R)$. (The Lichtenbaum conjecture at 0, because of the functional equation, amounts to the classical formula for the residue at $s = 1$ of the zeta-function, involving the regulator of $R^* = \mathrm{K}_1(R)$, the size of the torsion subgroup of R^*, and the class number of R.)

This division could be carried out explicitly in several cases by dividing b in the second Bloch group. The most notable case is that of $\mathrm{Q}(\sqrt{-303})$, where $\mathrm{K}_2(R)$ has order 22.

This is joint work with David Burns, Herbert Gangl, Alexander Rahm, and Dan Yasaki.