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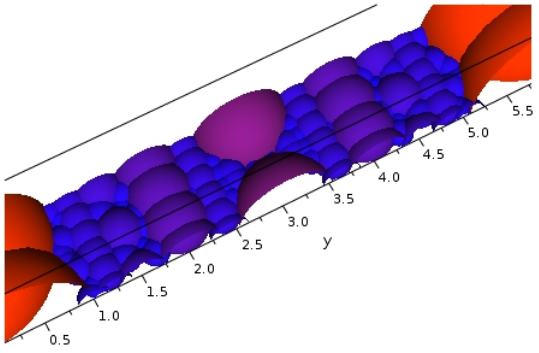
Experimental Mathematics Lab projects

Visualization of fundamental polyhedra in hyperbolic space

Introduction to the two projects:

The Bianchi groups are 2x2 matrix groups (e.g. $\mathrm{SL}_2(\mathbb{Z}[\sqrt{-m}])$ with m a natural number), and each of them acts on 3-dimensional hyperbolic space by pushing it forward and backwards in some directions, rotating it, and combining these movements. A fundamental polyhedron for this action is the selection of an area of hyperbolic space such that its copies under the movements by the Bianchi group together cover up all of hyperbolic space. We can require this area to be a polyhedron; and in order for it not to become too big, we can require that it touches its copies only at its facets. The shape of the Bianchi fundamental polyhedra is computed by the project supervisor's software *Bianchi.gp* in terms of hyperbolic coordinates, and has been visualized in the upper-half space model, where planes get distorted to hemispheres. A fundamental domain for $\mathrm{SL}_2(\mathbb{Z}[\sqrt{-37}])$, computed with *BianchiGP* and visualized by M. Fuchs in MuPAD, is shown here:



Project 1: 3-dimensional printing of the polyhedra. Taking a polyhedron from the output of *Bianchi.gp*, the polyhedron shall be triangularized:
Its 2-dimensional, polygonal facets shall be subdivided into triangles. Then it shall be fed into a 3D-printer, in order to create a 3-dimensional model in plastic.

Project 2: 3-dimensional visualization by rotations on screen. This project has the goal of visualizing the Bianchi fundamental polyhedra instead with the software *Geomview*, which provides hyperbolic camera views. Export of the hyperbolic coordinates from *Bianchi.gp* to *Geomview* is already implemented; so the start of the project will be easy, and it might be possible also to color the facets of

the polyhedron in pairs, which arise when there is a movement of one facet onto the other.

A visualization with geomview of a fundamental polyhedron (at discriminant -427) is shown here:

Data on these face identifications is readily availabe with *Bianchi.gp*, as well as data on more sophisticated aspects (such as the supports of homology generators) that can be visualized if the progress of the project allows. The latter data already yields 2-dimensional projections of the supports of homology generators.

The students will collect screenshots from *Geomview* and organise them in animation files (or develop a more practical export procedure).

Examples of Literature:

Geomview: `http://www.geomview.org/`

Keith Conrad, *Ideal classes and SL_2*,

`http://www.math.uconn.edu/~kconrad/blurbs/gradnumthy/SL2classno.pdf`

Requirements:

Algèbre linéaire, Structures mathématiques, Géometrie.

Language: English, French, or German.